Measurement 44 (2011) 1342–1348

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

A new process damping model for chatter vibration

Erol Turkes [a], Sezan Orak [b], Suleyman Neseli [c], Suleyman Yaldiz [c,*]

[a] Dumlupınar University, Department of Mechanical Engineering, Tavsanli Yolu 10. Km, Kutahya, Turkey
[b] Eskisehir Osmangazi University, Department of Mechanical Engineering, Bati Meselik, Eskisehir, Turkey
[c] Mechanical Department, Technical Science College, Selcuk University, 42031 Konya, Turkey

ARTICLE INFO

Article history:
Received 3 February 2011
Received in revised form 17 March 2011
Accepted 24 April 2011
Available online 12 May 2011

Keywords:
Analytical calculation
Chatter vibration
Process damping

ABSTRACT

This paper presents a new analytical process damping model (PDM) and calculation of Process Damping Ratios (PDR) for chatter vibration for low cutting speeds in turning operations. In this study a two degree of freedom complex dynamic model of turning with orthogonal cutting system is considered. The complex dynamic system consists of dynamic cutting system force model which is based on the shear angle (φ) oscillations and the penetration forces which are caused by the tool flank contact with the wavy surface. Depending on PDR, the dynamic equations of the cutting system are described by a new mathematical model. Variation and quantity of PDR are predicted by reverse running analytical calculation procedure of traditional Stability Lobe Diagrams (SLD). Developed mathematical model is performed theoretically for turning operations in this study and simulation results are verified experimentally by cutting tests.

1. Introduction

One of the most detrimental phenomena to productivity in metal turning operations is unstable cutting or chatter. Chatter can greatly reduce the life of tooling, dimensional accuracy, and the quality of a part's surface finish. Ideally, cutting conditions are chosen such that material removal is performed in a stable manner. Machine tool chatter vibrations result from a self-excitation mechanism in the generation of chip thickness during machining operations. One of the structural modes of the machine tool-workpiece system is excited by cutting forces initially. A wavy surface finish left during the previous revolution in turning, or by a previous tooth in milling, is removed during the succeeding revolution or tooth period, which also leaves a wavy surface owing to structural vibrations. Chatter is not desired, and it is formed independently from the machine tool and the outside environment. This self-excited type vibration occurs in metal cutting if the chip width is too large respect to the dynamic stiffness of the system. Under such conditions these vibrations start and quickly grow. The cutting force becomes periodically variable, reaching considerable amplitudes, and chip thickness varies in the extreme so much that it becomes dissected [1]. According to the literature, until today in the theoretical and experimental studies, process damping occurs due to penetration of the tool in the low cutting speeds. But, magnitude of the PDR and its effects to stability have not been studied. In these studies, the modeling of the cutting system is performed due to the acceptance of the fact that shear tool bit is constituted as a result of penetration to the rough surface of the piece. Penetration force and variations of cutting force that occurs from penetration into wavy workpiece surface of the tool have been studied [2–5]. Also, variable cutting force which occurs from shear angle (φ) variation because of oscillation of tool has been studied [6–9]. Comprehensive modeling of the dynamic cutting forces and, its effects in the chatter vibration have been studied [10–15] where dynamic cutting coefficients are obtained.

According to Tlusty [16], process damping force occurs in a section between tool cutting edge and wavy surface during the dynamic cutting. However, as it's not linear,

* Corresponding author. Tel.: +90 332 2232349; fax: +90 332 2410185.
E-mail addresses: eturkes@dpu.edu.tr (E. Turkes), sorak@ogu.edu.tr (S. Orak), sneseli@selcuk.edu.tr (S. Neseli), syaldiz@selcuk.edu.tr (S. Yaldiz).

0263-2241/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.measurement.2011.04.004

the modeling of the process is still arduous and the basic subject matter has not fully been comprehended though it is recognized why and how this force is produced. This is because fundamental aspects of this nonlinear process damping force are not fully understood. In spite of various investigations, a mathematical model is not still developed to define the relationship between clearance angle and process damping force. Furthermore, the mathematical equation used to describe this relationship has not yet been fully clarified. According to Lee et al. [5], the assumption that the process damping force is inversely proportional to the instantaneous clearance angle is over simplified. In addition to this, effects to structural damping of the PDR which occurs low and average cutting speeds have not been understood clearly because of realizing exactly why and how they are formed. But, damping modeling is necessary for the prediction of stability. Another point to be indicated here is that process damping is important also for the average cutting speeds depending on the vibration frequency, though it is generally seen with low speeds. Particularly, process damping which occurs from penetration into wavy surface of the tool is inversely proportional to wavelength (L_w). If the wavelength (L_w) is ratio to the chatter frequency (ω_c) of the cutting speed (V_c), process damping can occur for slightly higher cutting speeds (V_c) in spite of high chatter frequencies (ω_c). In high speed machining, stability lobes where higher stable depth of cuts are available can be utilized whereas, in low speed cutting, the process damping may have significant affect on stability. It is well known that higher stable cutting depths can be achieved under the effect of process damping. This can be important for increased productivity as low cutting speeds have to be used in many applications due to speed limitations on the machine, or for low machinability materials, such as titanium and nickel alloys which are commonly used in aerospace industry.

In this paper, a complex dynamic system is modeled prior to the orthogonal cutting, which forms a mechanical cutting basis for all cutting processes in general. This dynamic system is used for turning. In order to determine the increased values of the damping ratio, model analysis and chatter stability tests were performed for different materials, tool length and spindle speed. In this study, the influence of the cutting tool materials on our model is not considered. Because, according to the literature, the influence of the cutting tool materials is not important to determine Process Damping Ratios (PDR). Nevertheless, chatter stability at low cutting speeds, where the contact between the flank face of the tool and undulations left on the surface produces an additional damping effect, has not been fully modeled yet. Finally, the verification of developed dynamic cutting model is done not only for the shear angle oscillation coefficient, but also for the specific cutting resistance of workpiece in penetration. Furthermore, the most effective factors in the process damping are provided in [17].

2. Process damping model

In this section, process damping model (PDM) is modeled as dynamic orthogonal cutting process with the process damping force. According to constant the shear stress (τ_s) of workpiece material for various cutting speeds and feed ratios and the shear angle (φ) oscillations Static and Dynamic Cutting Force Coefficients (DCFC) are obtained by using the dynamic cutting force model. The constitution of DCFC is achieved through the research of how cutting mechanics and cutting forces alter under various cutting conditions. By employing the stability diagrams, the values of the total process damping corresponding to the stable cutting depth are calculated. The proportions of calculated PDR values due to the shear angle oscillations and the penetration forces which are caused by the tool flank contact with the wavy surface are determined. The prediction of maximum stable cutting depth requires knowledge of dynamic cutting coefficients characterizing relations between the dynamic cutting force and tool-workpiece vibration. The experimental determination of the DCFC requires sophisticated and expensive

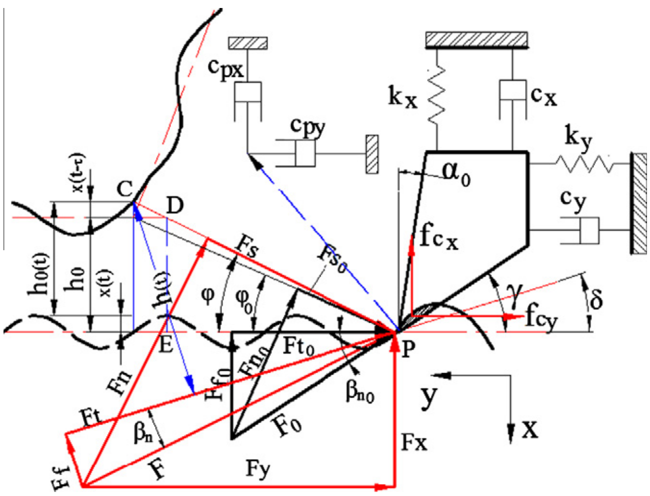

Fig. 1. Dynamic cutting model.

equipment used only in research laboratories. DCFC can also be obtained from steady-state cutting experiments in analytical modeling [9,13,14]. However, the experimental determination of DCFC requires considerable effort because of their dependence on the machining conditions [8,13]. In this study, a dynamic cutting model is developed similar to Kim & Lee [13] and Türkes [17]. This model consists of tool penetration effect and oscillation of tool effect as shown in Fig. 1.

2.1. Oscillation of cutting tool and establishing of DCFC

According to Fig. 1, without considering the penetration, the equations of motion of cutting tool in (x) and (y) directions are:

$$\begin{aligned} m_x\ddot{x}(t) + c_x\dot{x}(t) + k_x x(t) &= F(t)\sin(\beta_n + \delta) \\ m_y\ddot{y}(t) + c_y\dot{y}(t) + k_y y(t) &= F(t)\cos(\beta_n + \delta) \end{aligned} \tag{1}$$

where m_x and m_y are the mass coefficients (kg), c_x and c_y are the structural damping coefficients (kg/s), k_x and k_y are the stiffness coefficients (N/m). Dynamic force ($F(t)$ [N]) can be represented by:

$$F(t) = aK_f h(t) \tag{2}$$

where K_f is the feed force constant (N/m^2), and instantaneous dynamic chip thickness ($h(t)$) and dynamic chip thickness ($h_0(t)$) are:

$$h(t) = h_0(t) + |PE|\delta \rightarrow h_0(t) = h_0 + x(t-\tau) - x(t) \tag{3}$$

where (h_0) is undeformed chip thickness (m), ($x(t)$) and ($x(t-\tau)$) are modulations of the cut and outer work surface (m), (δ) is the instantaneous work surface slope generated by the tool oscillation being given by:

$$\begin{aligned} h(t) &= h_0 + x(t-\tau) - x(t) + \cot(\varphi)h_0 \quad \delta \rightarrow \delta \\ &= \tan^{-1}\left(\frac{\dot{x}}{V_0 + \dot{y}}\right) \end{aligned} \tag{4}$$

where (V_0) is the cutting speed of static cutting (m/s) and ($\dot{x}, \dot{y}$) are the vibration velocities (m/s) in each direction. For any given work and tool materials, it is always possible to establish an empirical relation between the shear angle (φ) and the mean friction coefficient (μ_a) through experimentation. A general form of the angle relationship [13,17,18] for machining steel is given by Wu & Liu [10]:

$$2\varphi_0 + \beta_{s0} - \alpha_0 = C \tag{5}$$

It is assumed that this relation between the shear angle (φ) and the friction angle (β_s) is invariant in dynamic cutting. Thus it can be represented by:

$$2\varphi + \beta_s - \alpha = C \tag{6}$$

where (C) is the material constant which is insensitive to the machining conditions. Also, ($_0$) sub-indexes in Eq. (5) are represented to static conditions. The relationship between the dynamic shear angle (φ) and relative cutting speed ($V = V_0 + \dot{y}$) can be represented by following equation:

$$\varphi = \varphi_0 + \eta_v\dot{y} \rightarrow \varphi_0 = \varphi_{01} + \eta_v V_0 \tag{7}$$

where (η_v) and (φ_{01}) are cutting constants for a given feed rate and tool rake angle. φ_0 is the average cutting angle acquired by V_0 cutting speeds of the test procedure above and by being applied for improvement rates. η_v is an improvement rate of an obvious cutting speed and dynamic cutting constant that changes according to the tool chip angle, while $\dot{y}$ is the speed of the tool in the direction of y. φ_{01} is an improvement rate of an obvious cutting speed and dynamic cutting constant that changes according to the tool chip angle. The η_v and φ_{01} constants of in these equations are calculated according to the principal of keeping the workpiece constant for various cutting speeds of the cutting tension and the improvement rates, by cutting tests. Writing the following formulas;

$$\eta_v = \frac{(\varphi_0 - \varphi_{01})}{V_0} \rightarrow \sin\varphi_{01} = \frac{ah_{01}\tau_s}{F_{s01}} \tag{8}$$

φ_{01} and η_v dynamic cutting constants are calculated. Making use of these formulas, the difference caused by the oscillation of φ, which is a theoretical and experimental cutting angle, is calculated. If this difference is substituted into the dynamic cutting model, dynamic resultant cutting force (F [N]) can be represented by:

$$\begin{aligned} F &= \frac{F_s}{\cos(\varphi + \beta_n + \delta)} = \frac{\tau_s a l_s}{\cos(\varphi + \beta_n + \delta)} \rightarrow l_s \\ &= \frac{h(t)}{\sin(\varphi + \delta)} \end{aligned} \tag{9}$$

where (l_s) is instantaneous shear plane length (m), (τ_s) is the shear stress of workpiece (N/m^2). Hence, dynamic cutting forces in both directions can be represented by:

$$F_x = F\sin(\beta_n + \delta); \quad F_y = F\cos(\beta_n + \delta) \tag{10}$$

Substituting Eqs. (6) and (7) into (10), and neglecting higher order terms of (δ) and ($\eta_v\dot{y}$) the following equations are obtained [17],

$$\begin{aligned} F_x &= \tau_s a\lambda_{sx}(h_0 - x + x(t-\tau) + \lambda_{dx}\dot{x} - \lambda_{vx}\dot{y}) \\ F_y &= \tau_s a\lambda_{sy}(h_0 - x + x(t-\tau) + \lambda_{dy}\dot{x} - \lambda_{vy}\dot{y}) \end{aligned} \tag{11}$$

where

$$\begin{aligned} \lambda_{sx} &= \frac{\sin(C - 2\phi_0)}{\sin\phi_0\cos(C - \phi_0)} \\ \lambda_{sy} &= \frac{\cos(C - 2\phi_0)}{\sin\phi_0\cos(C - \phi_0)} \end{aligned} \tag{12}$$

$$\begin{aligned} \lambda_{dx} &= \frac{h_0}{V_0}\left[\frac{\cos(\phi_0)\cos(C - \phi_0) - \cos(C)}{\sin(\phi_0)\cos(C - \phi_0)} + \frac{\cos(C - 2\phi_0)}{\sin(C - 2\phi_0)}\right] \\ \lambda_{dy} &= \frac{h_0}{V_0}\left[\frac{\cos(\phi_0)\cos(C - \phi_0) - \cos(C)}{\sin(\phi_0)\cos(C - \phi_0)} + \frac{\sin(C - 2\phi_0)}{\cos(C - 2\phi_0)}\right] \end{aligned} \tag{13}$$

$$\begin{aligned} \lambda_{vx} &= h_0\eta_v\left[\frac{\cos(C - 2\phi_0)}{\sin(\phi_0)\cos(C - \phi_0)} + \frac{2\cos(C - 2\phi_0)}{\sin(C - 2\phi_0)}\right] \\ \lambda_{vy} &= h_0\eta_v\left[\frac{\cos(C - 2\phi_0)}{\sin(\phi_0)\cos(C - \phi_0)} + \frac{2\sin(C - 2\phi_0)}{\cos(C - 2\phi_0)}\right] \end{aligned} \tag{14}$$

Thus the dynamic cutting force components have been expressed analytically by the static cutting coefficients $\lambda_{sx}, \lambda_{sy}$ and the dynamic cutting coefficients $\lambda_{dx}, \lambda_{dy}$ and $\lambda_{vx}, \lambda_{vy}$

which can be determined from static cutting tests. If DCFC used to the turning operations, PDM can be expressed as follows (1);

$$m_x\ddot{x}(t)+c_x\dot{x}(t)+k_x x(t)=-\tau_s a\lambda_{sx}(h_0-x+x(t-\tau)+\lambda_{dx}\dot{x}-\lambda_{vx}\dot{y})$$
$$m_y\ddot{y}(t)+c_y\dot{y}(t)+k_y y(t)=-\tau_s a\lambda_{sy}(h_0-x+x(t-\tau)+\lambda_{dy}\dot{x}-\lambda_{vy}\dot{y}) \quad (15)$$

Stability analysis of cutting system can be done by taking the Laplace transform and determinant of Eq. (15). Hence, the tangential cutting force coefficient (K_t [N/m^2]) and the feed force coefficient (K_f [N/m^2]) are defined as Türkes [17],

$$K_f=\tau_s\lambda_{sx};\quad K_t=\tau_s\lambda_{sy} \quad (16)$$

Thus, Eq. (15) are rewritten as follows;

$$m_x\ddot{x}(t)+c_{esx}\dot{x}(t)+k_x x(t)+c_{vy}\dot{y}(t)=F_{esx}(x(t)-x(t-\tau))$$
$$m_y\ddot{y}(t)+c_{esy}\dot{y}(t)+k_y y(t)+c_{dy}\dot{x}(t)=F_{esy}(x(t)-x(t-\tau)) \quad (17)$$

where

$$c_{esx}=c_x+aK_f\lambda_{dx};\quad c_{vy}=-aK_f\lambda_{vx};\quad F_{esx}=-aK_f$$
$$c_{esy}=c_y-aK_t\lambda_{vy};\quad c_{dy}=aK_t\lambda_{dy};\quad F_{esy}=-aK_t$$

2.2. Penetration of cutting tool

The process damping occurs further due to penetration of the tool into the wavy workpiece surface. A ploughing force model has been proposed to describe the contact relationship between tool flank and machined surface. Based on this ploughing force analysis, a small volume of workpiece material is pressed by the tool during wave cutting. In the meantime, a resistance force is generated by the stress field inside of the displaced workpiece material. It is therefore reasonable to assume that the process damping force is equal to resistance force. The resistance force has been proven to be proportional to the volume of the displaced workpiece material [5,16]. Hence, the equations of motion in the (x) and (y) directions are written as follows:

$$m_x\ddot{x}(t)+c_x\dot{x}(t)+k_x x(t)=F(t)\sin(\beta_n+\delta)=-F_{xTop}(t)$$
$$m_y\ddot{y}(t)+c_y\dot{y}(t)+k_y y(t)=F(t)\cos(\beta_n+\delta)=-F_{yTop}(t) \quad (18)$$

total dynamic forces in both directions are written as follows:

$$F_{xTop}(t)=-(F_x(t)+f_{cx}(t))$$
$$F_{yTop}(t)=-(F_y(t)+f_{cy}(t)) \quad (19)$$

where $f_{cx}(t)$ and $f_{cy}(t)$ are resistance force components in the (x) and (y) directions (N) and they can be expressed as:

$$f_{cx}(t)=c_{px}\dot{x}(t);\quad f_{cy}(t)=c_{py}\dot{y}(t) \quad (20)$$

Hence, the equations of motion in the (x) and (y) directions are written as:

$$m_x\ddot{x}(t)+c_{tpx}\dot{x}(t)+k_x x(t)=-F_x(t)$$
$$m_y\ddot{y}(t)+c_{tpy}\dot{y}(t)+k_y y(t)=-F_y(t) \quad (21)$$

where (c_{tpx}) and (c_{tpy}) are the total penetration damping coefficients (kg/s) and they are defined as

$$c_{tpx}=c_x+c_{px};\quad c_{tpy}=c_y+c_{py} \quad (22)$$

Hence, due to tool penetration effect and oscillation of tool effect (21) equations are written as,

$$m_x\ddot{x}(t)+c_{tsx}\dot{x}(t)+k_x x(t)+c_{vy}\dot{y}(t)=F_{esx}(x(t)-x(t-\tau))$$
$$m_y\ddot{y}(t)+c_{tsy}\dot{y}(t)+k_y y(t)+c_{dy}\dot{x}(t)=F_{esy}(x(t)-x(t-\tau)) \quad (23)$$

where (c_{tsx}) and (c_{tsy}) are total damping coefficients in the both directions, and written as

$$c_{tsx}=c_{tx}+aK_f\lambda_{dx};c_{tsy}=c_{ty}-aK_t\lambda_{vy};c_{vy}=-aK_f\lambda_{vx};c_{dy}=aK_t\lambda_{dy};\quad F_{esx}=-aK_f;\quad F_{esy}=-aK_t$$

For calculation of the (c_{px}) and (c_{py}), to properly calculate the volume of the workpiece material pressed by the tool, the displaced volume is sliced into many segments as in Fig. 2 [5]. Where (n), is the number of segments, (γ) is the tool clearance angle, (α_0) is the chip flow angle, (A) is the total displaced area of work material (m^2), (A_k) is the k th displaced segment area which can be calculated as:

$$A_k=\left(\frac{\Delta x_{i-k}+\Delta x_{i-k-1}}{2}\right)\Delta y_k;\quad V_h=aA_k=a\sum_{k=1}^{n}A_k \quad (24)$$

and where

$$\Delta x_{i-k}=(x_{i-k}-x_i)-k\Delta x_k\tan\gamma;\quad \Delta x_{i-k-1}=(x_{i-k-1}-x_i)-(k+1)\Delta x_k\tan\gamma \quad (25)$$

The resistance force components $f_{cx}(t)$ and $f_{cy}(t)$ in the (x) and (y) directions can be expressed as [5,17];

$$f_{cx}=f_{sp}V_h;\quad f_{cy}=f_{cx}\mu_c \quad (26)$$

where (f_{sp}) is the specific resistance (N/m^3), (V_h) is the volume of the ploughing to wavy surface of cutting tool (m^3), (μ_c) is the mean friction coefficient. Hence, stability analysis of the cutting system can be predicted by the Eq. (26).

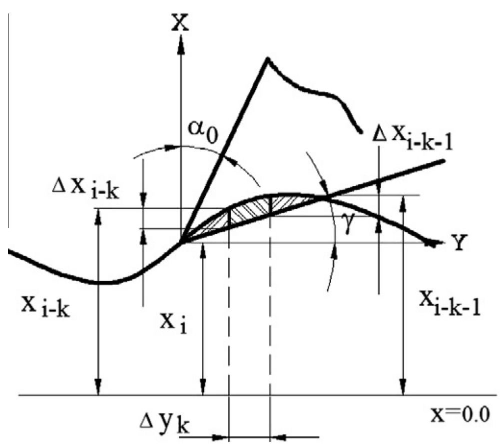

Fig. 2. Penetration model of cutting tool.

This prediction is achieved by running to start from end of the conventional stability analysis procedure [17].

3. Obtaining total process damping ratios

Model analysis of the cutting system is performed by impact force hammer set and CutPro 8 software. Material of workpiece is AISI-1010 ($K_f = 1 \times 10^{+9}$ N/m^2), cross-section dimensions of cutting tool are (20 × 20) mm and attach length of tool is L = 70; 90; 110 mm. Chatter frequencies (ω_c [Hz]) in Table 1 have been obtained by LabView 7.1 software with cutting tests. Experimental model analysis of cutting system has been performed by measurement of transfer functions using a hammer instrument with a force transducer and an accelerometer attached to machine tool structure. Where, we have used an accelerometer with measurement range ± 50 g, sensitive 104.3 mV/g, resonant frequency 40.0 kHz and an impact force hammer with force range 0–500 N, sensitive 10 mV/N impulse. As an example, the results of modal analysis for the attach length of cutting tool L = 70 mm are given in Fig. 3. This figure is transfer function diagram of dynamic cutting system with L = 70 mm. In Fig. 3, the second mode is dominant mode of cutting system.

In order to determine the increased values of the damping ratio, modal analysis and chatter stability tests were performed for different materials, tool length and spindle speed. By employing the stability diagrams, the values of the total process damping corresponding to the stable depth of cuts are calculated. Stable critical axial cutting depths ($a_{\lim}$ [mm]) were determined by gradually increasing axial cutting depths during the dynamic cutting tests. At the same time, chatter frequency (ω_c) of the cutting system was determined by microphone test. Hence, chatter frequency is determined by voice frequency of the cutting system, which is a process with LabView 7.1 software [17].

Table 1
Outputs of the impact hammer and microphone tests.

Tool length, L (mm)	Natural frequency, ω_n (Hz)	Stiffness, k (N/m)	Damping ratio, ζ (%)	Chatter frequency, ω_c (Hz)
70	1644	2.24×10^7	2.56×10^{-2}	1680
90	1055	8.39×10^6	2.67×10^{-2}	1040
110	801.3	4.76×10^6	1.36×10^{-2}	880

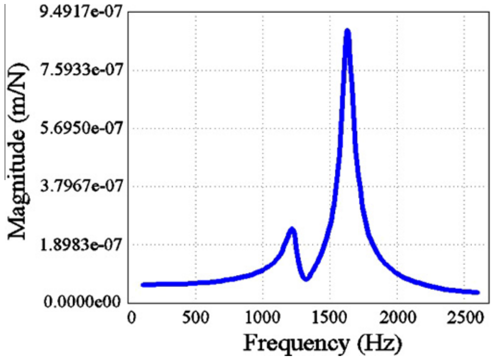

Fig. 3. Transfer function diagram of dynamic cutting system with L = 70 mm.

Where (L [mm]) is the attach length of cutting tool, (ζ [%]) is the damping ratio, (ω_n [Hz]) is the natural frequency, (k [N/m]) is the stiffness coefficient. The critical axial depth of cut ($a_{\lim}$ [mm]) in Table 2 has been determined by the gradual increase of the axial cut depth and by perform of the cutting tests with low cutting speeds or low spindle speeds (n [rpm]). Feed rate in cutting tests is s = 0.06 mm/rev and constant.

The total damping ratio (ζ_T [%]) of the cutting system can be expressed as:

$$\zeta_T = \zeta_{str.} + \zeta_{s\varphi} + \zeta_{sp} \tag{29}$$

where ($\zeta_{str.}$) is the structure damping ratio, ($\zeta_s\varphi$) occurs from the shear angle variation, (ζ_{sp}) occurs from penetration to wave surface of the tool. Determination of the total damping ratios in the different spindle speeds and critical axial depths of cutting for turning and milling operations is achieved by running to start from the end of the conventional stability analysis procedure as in [1,19,20]. Hence,

$$a_{\lim} = \frac{-1}{2K_f G_{\min}(\omega)} \tag{30}$$

where ($a_{\lim}$) is the stable cutting depth and it is determined from cutting tests in the low spindle speeds. Therefore, area which is under the asymptotic curve is stable cutting area. This case has been showed in Fig. 4a.

Stability Diagrams (SD) in Fig. 4b, Fig. 5a and b is plotted according to values in Tables 2 and 3. The cutting depths on the lowermost points of curve ($a_{\lim}$) in Fig. 4a are corresponded to real part ($G_{\min}$ [m/N]) of the transfer function of cutting system. Each one of the ($G_{\min}$) points can be expressed as:

$$G_{i\min}(\omega) = \frac{-1}{4k_i\zeta_i(1+\zeta_i)} \tag{31}$$

Stable cutting depths that are obtained by cutting tests and chatter frequency are obtained according to dominant mode of the cutting system. Therefore, cutting system is accepted as Single Degree of Freedom (SDOF) and how to change the system damping ratio can be examined. Where the dominant mode is in the direction (x) and therefore damping ratios are estimated by quadratic equation,

$$\zeta_x^2 + \zeta_x + \frac{1}{4k_x G_{\min}(\omega)} = 0 \tag{32}$$

After the determination of the stable cutting depths for different spindle speeds and real part (G [m/N]) of the transfer function imaginer part (H [m/N]) of the transfer function is

Table 2
Determined ($a_{\lim}$ [mm]) values by cutting tests.

Tool length, L (mm)	Spindle speed, n (rpm)						
	90	125	180	250	355	500	710
70	7.8	7.4	6.7	6.0	5.3	4.5	3.8
90	6.5	6.0	5.4	5.0	4.5	3.7	3.2
110	5.1	4.7	4.2	3.8	3.5	3.0	2.5

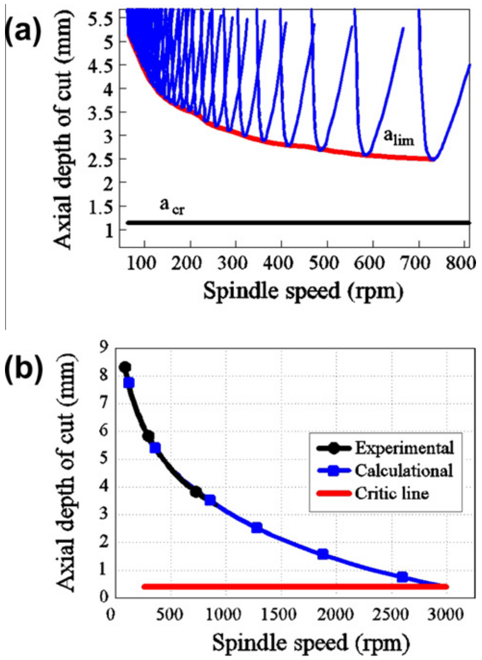

Fig. 4. (a) Cutting depth with stability lobes and (b) SD for L = 70 mm tool length.

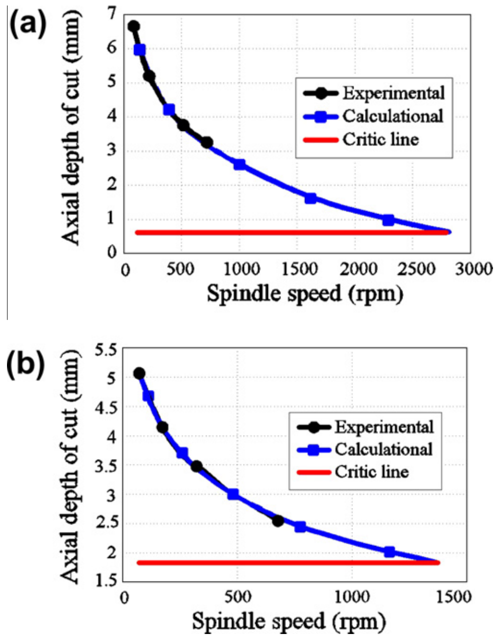

Fig. 5. (a) L = 90 mm and (b) SD for L = 110 mm tool length.

Table 3
PDR (ζ_{sT} [%]) values for the AISI-1010 material.

Tool length, L (mm)	Spindle speed, n (rpm)						
	90	125	180	250	355	500	710
70	0.1650	0.1600	0.1410	0.1260	0.1100	0.0913	0.0750
90	0.3414	0.3200	0.2910	0.2720	0.2470	0.2100	0.1800
110	0.4600	0.4310	0.3940	0.3633	0.3400	0.3000	0.2554

obtained by reverse running analytical calculation procedure of traditional SLD. Imaginary part of the cutting system is calculated by Eq. (33).

$$H(\omega_t) = G(\omega_t)\tan(\psi) \quad (33)$$

4. Conclusions

In this study, a new analytical PDM is developed for the determination of the PDR. This PDR occurring in low and middle cutting speeds. This new analytical PDM gives quite reliable results and it is a fairly easy method. Besides, this model provides a new approach for the calculation of PDR problem which has been unsolved for years. These PDR obtained by this model are completely realistic values. Because, stable $a_{\lim}$ values which obtained by dynamic cutting tests are used for the analytical model investigated here. Hence, results of the analytical PDM and dynamic cutting tests give the same diagrams or asymptotic curves as shown in Figs. 4 and 5. The advantage of this PDM than other models is that it can be used for different cutting conditions, workpiece materials and tool/tool holder constructions. Thus, PDR could be predicted according to cutting conditions, workpiece material and tool/tool holder properties. Also, if cutting tests performs for enough numbers, the prediction ability of the PDR values will be achieved during the model analysis of cutting system.

Acknowledgments

Laboratory Works in this study are performed correspondingly by the studies of the research project of Scientific Research Project Commission of the Eskişehir Osmangazi University, entitled "Stability Analysis of Chatter Vibration on Machine Tools" and numbered 200615012, in Turkey.

References

[1] Y. Altintas, Manufacturing automation; metal cutting mechanics, machine tool vibrations and CNC design, Cambridge University Press, New York, 2000.
[2] P.W. Wallace, C. Andrew, Machining forces: some effects of tool vibration, J. Mech. Eng. Sci. 7 (1965) 152.
[3] P.W. Wallace, C. Andrew, Machining forces: Some effects of removing a wavy surface, J. Mech. Eng. Sci. 8 (1965) 129.
[4] J. Tlusty, Analysis of the state of research in cutting dynamics, Man. Tech. CIRP Ann. 27 (1978) 583.

[5] B.Y. Lee, Y.S. Tarng, S.C. Ma, Modeling of the process damping force in chatter vibration, Int. J. Mach. Tools Manufact. 35 (1995) 951.
[6] W.A. Knight, Some observations on the vibratory metal cutting process employing high speed photography, Int. J. Mach. Tool Des. Res. 10 (1970) 221.
[7] J. Tlusty, T. Moriwaki, B.S. Goel, The dynamic cutting force coefficients for some carbon steels, in: Proc. 4th NAMR Conference SME, 1976, p. 287.
[8] M.M. Nigm, M.M. Sadek, S.A. Tobias, Determination of dynamic cutting coefficients from steady state cutting data, Int. J. Mach. Tool Des. Res. 17 (1977) 19.
[9] K. Srinivasan, C.L. Nachtigal, Investigation of the cutting process dynamics in turning operations, J. Eng. Ind. Trans. SME 1 (1978) 77.
[10] D.W. Wu, C.R. Liu, An analytical model of cutting dynamics, part 1: model building, J. Eng. Ind. Trans. ASME 107 (1985) 107.
[11] D.W. Wu, C.R. Liu, An analytical model of cutting dynamics, part 2: verification, J. Eng. Ind. Trans. ASME 107 (1985) 112.
[12] D.W. Wu, Comprehensive dynamic cutting force model and its appllication to wave-removing process, J. Eng. Ind. Trans. ASME 110 (1988) 153.
[13] J.S. Kim, B.H. Lee, An analytical model of dynamic cutting forces in chatter vibration, Int. J. Mach. Tool Des. Res. 31 (1991) 371.
[14] K. Jemielniak, Modelling of dynamic cutting coefficients in three-dimensional cutting, Int. J. Mach. Tool Des. Res. 32 (1992) 509.
[15] M. Xiao, S. Karube, T. Soutome, K. Sato, Analysis of chatter suppression in vibration cutting, Int. J. Mach. Tools Manufact. 42 (2002) 1677.
[16] G. Tlusty, Manufacturing Processes and Equipment, Prentice Hall, New Jersey, 2000.
[17] E. Türkes, Theoretical and Experimental Analysis of Process Damping in Machine Tool Chatter Vibration, Ph.D. Thesis, University of Eskisehir Osmangazi, Turkey, 2007.
[18] J.A. Bailey, Friction in metal machining-mechanical aspects, Wear 31 (1975) 243.
[19] Y. Altintas, E. Budak, Analytical prediction of stability lobes in milling, Ann. CIRP 44 (1995) 357.
[20] E. Budak, Y. Altintas, Analytical prediction of stability lobes in milling part i: general formulation, part ii: application of the general formulation to common milling systems, Trans. ASME J. Dynam. Syst., Measur. Control 120 (1998) 22.