An Analysis of Students' Communication during Group Work in Mathematics

Pinelopi D. Vasileiadou[1]

Author Note

This study is my research project for my undergraduate degree in School of Education, Aristotle University of Thessaloniki, which was developed under the supervision of my professor Despoina Desli, Assistant Professor, Faculty of Education, Aristotle University of Thessaloniki.

ABSTRACT

Utilizing observation grid and interview study methologies, this research examines the ways in which students communicate with each other while working in a team during problem solving in mathematics. The study focuses primarily on the language used for communication. Results suggest that participants make assumptions to solve mathematical problems and justify their individual opinions, and cooperate and help each other, rarely asking for their teacher's help, while using both the ordinary, and the mathematical spoken and written language. The interview indicates that students, although not experienced in undertaking group work, are able to readily identify its benefits and positive aspects.

Keywords: Mathematics education; cooperative learning; problem solving; communication; language;

INTRODUCTION

Mathematics is the science that has as an object the systematic examination of physical sizes, shapes, signs, numbers and their relationships. It is a human activity based both on the active participation of the individual and coordinated communication among the members of a community. Therefore, students are not passive recipients of knowledge from mathematics' teachers, and are not obliged to mechanically reproduce this knowledge. At the same time, they are able to be led to the construction of their own mathematical meanings through the solution of problems of their own and their interaction with other members of the class (Bishop 1985).

While solving a problem in mathematics, each student follows a thought process leading to a solution. However, this process takes another meaning in peer collaboration. Here, the student must reciprocally share his or her thought processes with other members of his team. In so doing, the thought processes of all group members are impacted. The interaction of the thinking strategies students use within the group is a very important step for the development of mathematical concepts and provides learning opportunities which cannot be found either in 'traditional' teachings or in individual problem solving. One question we might ask here is how the original train of thought is affected. That is, what influences the thoughts of each member? Is it the spoken language, the written language, a combination of the two, or perhaps some other way? In other words, how do students communicate with each other in problem solving and how does this ultimately impact the solution of each problem?

Consequently, communication is essential to building new mathematical knowledge, and for the quality of this knowledge. More specifically, communication provides students the opportunity to construct mathematical knowledge through: efforts to settle differences when opinions conflict (Perret - Clermont 1980), efforts to distance themselves from their own activity in order to understand an alternative interpretation or solution (Sigel 1981), as well as efforts to reconstruct a mathematical idea or solution in order to be able to express it (Levina 1981).

[1] Pinelopi D. Vasileiadou, Teacher of Primary Education, Greece. penevasi@gmail.com

Pirie (1998) reports that the methods of communication are varied, and include ordinary language, mathematical verbal language, symbolic language, visual representations, unspoken but shared assumptions, and quasi-mathematical language.

This study examines the way Greek students communicate within a group whilst attempting to solve a mathematics problem. The goal was to investigate the participation and action of each child, their communication with each other and with their teacher, and the role of the group and the language used. The following research questions were developed to explore this goal: how do students participate and act in problem solving during group work? How does a student communicate with the other team members? What is the role of the group in problem solving? How does the group communicate with the teacher? What kind of language does the group use for communication?

LITERATURE REVIEW

Peer collaboration in Mathematics teaching and problem solving

Cooperative learning is a pedagogical teaching method in which students work together in groups of two to five members and each member participates in solving a common problem, without the direct intervention of a teacher. These groups are not random, but they are determined by the teacher. The teacher takes into account several parameters in order to create productive teams. These parameters are determined by each lesson and the goals the teacher sets (Chionidou-Moskofoglou, 2000. Matsagouras, 2001).

In the traditional classroom, the lesson is based on lectures and the center of the learning process is the teacher. In such an environment, students are only passive receivers. Their only action is to record knowledge when they watch the lecture of their teacher or when the teaching process takes the form of questioning. As a matter of fact, in the latter form, only correct answers are accepted, while the incorrect answers are ignored (Effandi & Zanaton, 2007). This process can often be lonely and frustrating for students. Perhaps it is not surprising that many students and adults 'fear' mathematics. They often believe only a few talented people can achieve in the field of mathematics (Davidson, 1990).

Communication in the Mathematics classroom

According to the online Oxford dictionary[2], communication is the impart or exchange of information by speaking, writing, or using some other medium. It is a multifaceted mechanism that impacts all aspects of society and has a close relationship with language.

In recent years, many researchers in the area of mathematics teaching have shown particular interest in studying communication in the classroom (Bartolini Bussi, 1991. Bauersfeld, 1988. Cobb, 1986. Cobb, Wood & Yackel, 1991a. Voigt, 1985). This interest is directly linked to the increasingly wide acceptance of constructional philosophy as a frame of reference for the study and understanding of issues related to the nature of mathematics, learning mathematics and teaching mathematics (von Glasersfeld, 1987, 1990).

The utility of communication is not only limited to the emergence of mathematical meanings individually derived by students, but also to the conditions for the development of a field of consensus for negotiating these meanings with other members of the class. Therefore, communication ensures the attainment of intersubjectivity, which forms the basis for meaningful conversation and not just an exchange of ideas. It should be emphasized, however, that amendment of obligations and expectations which apply to the traditional classroom for both students and teacher are prerequisite to this (Cobb, Wood, & Yackel, 1991b). More specifically, in an optimal environment, students should be able to explain and justify their thoughts, should not be afraid to make mistakes, respect their peers' opinions, and not consider the teacher as a source of mathematical authority.

[2] http://oxforddictionaries.com/definition/communication retrieved 3/23/2012

In summary, classroom communication is an activity requiring the active participation of its members and ensures the creation of a field of consensus on the negotiation of mathematical meanings. This also can be concluded from the research of Yackel, Cobb, Wood, Wheatley & Merkel (1990), who indicate that mathematics learning is an active process of problem solving. When you give students the opportunity to express how they understand mathematics, then potential problems of communication are highlighted. These problems, and the same mathematical operations, are opportunities for learning mathematics. However, Curcio (1990) and Pirie (1998) suggest that communication of mathematical ideas is legitimate to be related to the experiences of students in different grades.

Language and Mathematics

The literature demonstrates the interrelationship between language and mathematics. This relationship can be interpreted in two mutually-dependent ways: "*The language can be seen as an integral and important part of building mathematical knowledge or mathematics can be considered eminently linguistic activity"* (Sakonidis 1999 p. 455).

Pirie (1998) reports that the methods of communication are varied, and include ordinary language, mathematical verbal language, symbolic language, visual representations, unspoken but shared assumptions, and quasi-mathematical language.

Here, the term *ordinary* suggests the use of everyday vocabulary from each student, which of course, varies depending on the respective age and level of understanding of each student. The *mathematics verbal language* includes the use of words both in oral and in written language. *The symbolic language* is the kind of communication which is accomplished through written mathematical symbols.

Visual representations, even though not a form of language, is nonetheless a very powerful and useful tool for communication in mathematics. The *unspoken but shared assumptions* also do not fit exactly within a traditional definition of language, but are a necessary precursor for mathematical understanding.

Finally, *quasi-mathematical language* is often used when no mathematical language is directly available, when, for example, language is too sophisticated for the student, or when a metaphorical representation is obtained and translated in a strictly literal manner. This is a practice used by all who communicate with mathematic terminology, albeit with varying levels of acceptance in different contexts. When utilizing quasi-mathematical language, students often create new words, which do not exist in their pre-existing vocabulary to facilitate communication within the group in order to solve a problem.

Other researchers refer only to the oral language for communication in mathematics. Regarding to oral communications within problem solving groups, Kotsopoulos (2010), in a survey of 12-year old students, notes that these students talk aloud during problem solving in mathematics. This talking aloud serves three primary purposes: "*clarification of thinking, expressions of confusion, and a combination of clarifications of thought and expressions of confusion with the explicit intent of eliciting support from peers*" (p. 1065).

The plethora of means of communication creates problems in understanding of the mathematical concepts that students build. An example given by Pirie (1998) is that mathematics has a particular communication problem resulting from the fact that the language used when students speak about mathematics, differs from the one used when students write mathematics (as opposed to when they write *about* mathematics).

Strict mathematical language depends almost entirely on its unique symbols and technical terminology. In order for students to succeed in the study of mathematics, understanding it in depth, they should be allowed to explore, examine, and express mathematical relationships with which they come into contact in their everyday life, rather than to be asked to memorize official definitions and symbolisms absent relevant everyday context (Curcio, 1990).

Curcio (1990) argues that listening, speaking, reading and writing help students to clarify their ideas and share them with the others. Therefore, a natural way to help students develop their mathematical language is a language-experience approach in the teaching of mathematics.

Using such an approach, students have the opportunity to use everyday language to express ideas about mathematical concepts presented in a context of a situation from everyday life, to clarify their thinking, and to ponder the mathematical ideas they encounter. The approach of linguistic experience can help to bridge the gap between mathematics of the real world and school mathematics (Curcio, 1990).

The studies identified in this first chapter, regarding communication between students undertaking group problem solving in mathematics, relate only to students aged 12 and up, and can only be found in foreign language literature (not Greek). No Greek research was found relating to even younger ages, such as students in elementary school. The research that follows aims to cover this gap, drawing upon the increasing use of peer collaboration in Greek primary schools.

METHOD

Participants

For the purposes of this study, the first step was to find a class that students were at least familiar with separation into groups to solve problems. An optimal case would be where students were being taught with the peer collaboration method. After consultation, two primary schools in Thessaloniki agreed to cooperate. The first is located in the district of Ambelokipi (School A), while the second is in the district of Pilea (School B). The sixth grade of School A, and the fifth grade of School B, agreed to participate.
The groups, and the students, who participated in the research, were randomly selected. In School A two students were observed: a boy (Boy 1) and a girl (Girl 1). Two students were also observed from School B: a boy (Boy 2) and a girl (Girl 2). Boy 1 was a student of modest level, while Girl 1 of medium-low level. In School B both Boy 2 and Girl 2 were students of high level. The group observed from School A consisted of five members, three girls and two boys. The team from School B consisted of four members, one girl and three boys.

Research tools

Data collection involved two methods: an observation grid specifically designed for the purpose of the research, and a subsequent interview of the students who participated in the survey. The observation grid consists of 29 observational data which are divided into five sections: a. participation and activity of the child in problem solving, b. communication within the group, c. the role of the group, d. communication between students and teacher, and e. the use of language in communication. The observation grid is attached at Appendix I.

During group observations, the observer also kept handwritten notes on procedures and dialogues that occurred during problem solving.

The subsequent interview took place after the second day of observation at each school. At the break, after the course of mathematics, students were asked to answer open-ended questions about peer collaboration. The questions were put forward while both students were present, and the answers were given by each child separately. The questions are:

Interview questions:
- Do you enjoy working in a team or you prefer to work alone?
- Why?
- What was it that you liked most?
- What makes it difficult while working in a group?
- What do you think you learnt by working in a team?

Procedure

Data collection involved observing groups of two mathematics classes for two school hours each, over two consecutive days. I was in the classroom, and only observed without interfering with the work.

The interview took place with both students present. It occurred during the students' break, after mathematics class, on the second day of observation.

RESULTS

School A

The 6th grade and the Mathematics class

On both observation days, the classroom climate was pleasant. Naturally there was activity, but the students were listening to the teacher and all of them participated in the activities. The students were divided in three groups: two groups of five students and one of four. The teacher on the first day before the class of the observation, made some changes in the composition of the teams, repositioning and changing group two students, while the second day when we entered the class there were two groups of six to seven students and before we start the observation, the teacher changed their position to form the three groups of the previous day. As for the class of mathematics, on both days of observation, the teacher didn't use the blackboard or any mathematics textbook. As a matter of fact, both days the students were revising problems they had previously learnt.

1st day of observation

Course flow:

The teacher distributed three leaflets to the students. The first contained instructions on how to solve problems working as a team. The second contained five problems about the chapter's revision and the third was different for each child and asked each child to write the solution of the problem they would choose. The problems were related to the concept of proportion.

First, the students chose the name for their team. Then, each chose a problem. The student, who had the first problem, read it out loud for the whole group to listen to. After that, each child had to solve the problem alone. When they all had finished, they announced the solution they had reached, and the student who had the problem, wrote the solution reached on the third leaflet. The same process was repeated for each problem.

Student participation and activity

In this category of the observational data, Boy 1 had the greater involvement. Initially, he participated in all four problems the group solved, with quite an active role. Similarly, Girl 1 participated in solving all four problems but with quite a passive role. Boy 1 frequently made assumptions for the solution of the problems, while Girl 1 was not involved at this stage. Still, Boy 1 rarely substantiated his thinking, instead often expressing his opinion, both for the way of solving a problem, and for the mathematical operations needed to be done in order to find the solution. Illustrative is the example that follows.

(Boy 1 tries to help Girl 1)

Boy 1: You used 'proportion'. I did "crossed". Wait to do the verification.[3]

Within the group, Boy 1 seemed to be fairly competitive. On the other hand, Girl 1 rarely expressed her opinion, and preferred to remain silent during problem solving.

More generally, Boy 1 was expressive and spoke often. As presented below, his contributions started to confuse the minds of other team members.

Girl 1: I don't understand. With what did you do this?

(They do the multiplication together again (boy 1 and girl 1). They both ended up confused).

Boy 1 read aloud the third problem. He didn't understand the problem and he read it again. He still could not understand the question. The whole team became confused. Another member of the group then intervened and clarified the question.

In addition, Boy 1 had often finished solving a problem, while the rest of the team was still working, and he started alone on the next problem. At other times, he was discussing irrelevant things with other members of the group who had also finished solving their problems.

[3] With "proportion" is stated the equation a/b=c/d. With "crossed" is stated the process students learn to use to go from a/b=c/d to a*d=b*c. The two equations are equal.

Boy 1 started to read the next problem before the other members had a chance to.
Girl 1: Wait! (more members complained) You have to listen to the others!
Boy 1 stopped reading the next problem and asked the other group members what the results of their multiplication calculations were.

The general climate of the team was more akin to Girl 1 in the observation grid. The other members participated in the solution of the problems, but they did not make any suggestions for the solutions, did not justify their thoughts, were not competitive, did not interrupt the other team members, did not occupy themselves with irrelevant things, and did not abandon any activity. Two group members remained silent throughout the duration of the course, and spoke only when addressed (rarely), and also rarely expressed their opinion.

Communication within the group
Communication and collaboration between team members could be described as very good.

Boy 1, according to the observation grid, worked 7 times with other members of the team, especially on the process of resolving the problems. However, sometimes he cooperated only with one other student, ignoring the others. The following abstract is suggestive.
Girl 1: (addressing to Boy 1 and the other member with who he cooperates) Hey, guys, work as a group! It’s not only you two!
He quite often encouraged the other members to solve the problem. Less frequently, he asked others to support his thoughts, but most often only sought confirmation from other members. In the observer’s opinion, the confirmation he sought was not to see if he had reached a correct results, but to show that he had found the right solution, in other words to show off to the other members. The same is true for the queries he expressed to other team members. Very often he expressed his opinion on observations made by other members of the team, but he also made several comments to his classmates directing what they should do. Finally, quite often he completed the thoughts of his peers.
Girl 1 on the other hand, worked less with the other members of the group, relative to Boy 1, and sought support of her thought processes and problem solving methods regularly. Finally, she often expressed queries.

The role of the group
The group, as stated above, worked quite well. In all cases the team solved the queries of the other members. The conversation rarely was heated or intense, with only two instances of disagreement. The first time, was due to an individual’s misunderstanding. The second time, the disagreement was greater, and resulted in the team sidelining the member who started the dispute. In both disputes Boy 1 under observation had a very active role. Exemplary is the dialogue that follows:

Girl 1: How much did you find? (Addresses to the boy)
Boy 1: I found "that much".
Girl 1: You're wrong! It is "so much".
Boy 1: Why should I be wrong? And how do you know that I made the mistake?!
Girl 1: Very well … We will see how much the others are going to find.
(the other members find another result and actually Boy 1 was wrong).

Communication between students and teacher
Coming back to the two students of the observation, Boy 1 asked three times for clarifications from his teacher about the way in which the problems had been expressed. He twice asked for help, and twice raised questions about the correct way of solving a problem, and the outcome of another.

Girl 1 asked three times for clarifications from the teacher, and asked the team a question and asked for help once, respectively.
Two of the remaining members of the team did not raise queries to the teacher. The last remaining team member asked the teacher for one clarification.

The use of language in communication

This section was a bit hard to study because of the nature of the problems the group had to solve. However, it was observed that the two research subjects and the rest of the team used "ordinary language", "mathematical written language" and, albeit very rarely, "silent but common assumptions".

2nd day of observation

Course flow:

When the lesson started, the teacher shared with the students a corrected version of the problems they had solved in the last lesson, and each group announces its solutions to the class. Once the procedure was concluded for all the problems, the teacher distributed new problems which once again related to the notion of proportion, and indicated that the students should follow the same procedure as on the previous day.

Participation and activity of the student in problem's solution

Both Boy 1 and Girl 1 participated in the solution of all three problems. The group employed many of the same assumptions as on the previous day, to find solutions to the problems. Boy 1 justified his thinking and expressed his opinion more often than Girl 1.

Communication within the group

Girl 1 worked more with the other members of the team, than Boy 1. However, Boy 1 seemed to encourage his peers quite often, while Girl 1 did not. Boy 1 and Girl 1 regularly expressed their questions to other members of their group.

Boy 1 expressed, on a number of occasions, his opinion on observations made by other team members. He also completed, almost all the time, the sentences of his peers.

Furthermore, Girl 1 often asked for support and confirmation of her thought processes from the other members of the group, more so than Boy 1 did.

The role of the group

The team solved all the queries of the other members, despite their large number. Intense debate occurred. During most problems, members came to a disagreement. The first time, they disagreed because Girl 1 told Boy 1 to cooperate with other members of their team and not just with one member, Tanya. In another instance, Boy 1 made a mistake and disagreed with Tanya, until he realized that he had made a mistake and apologized.

Communication between students and teacher

Boy 1 requested clarifications from the teacher about the phrasing of the problems, and several times expressed his opinion on the comment made by the teacher. He also raised two queries with the teacher, and asked the teacher for help once.

The use of language in communication

The team quite often used the "ordinary language", and, more specifically, Boy 1 and Girl 1 of the observation. Only once did Boy 1 employ the use of "oral mathematical language". The use of "mathematical written language" was even more common than the use of "ordinary language", both from Girl 1 and Boy 1. Finally, only once did Girl 1 use a "silent but common assumption."

General notifications

Of particular interest is the fact that on this second day of observation Girl 1 spoke out loud to herself in an attempt to resolve the problem, similar to what Kotsopoulos (2010) studied in her research. She was talking aloud mathematical operations she had done while writing them down. It seemed that none of the other group members considered it atypical, and they didn't give her any attention when she did so.

Moreover, on the second day, the teacher assisted too much in the work of the team, so much so that she was helping some students solve problems, which allowed them to move on to the next problem, but left the rest of the team behind. With this intervention she disorientated the group's work. The members talked to each other mainly about the results of the solutions of the problems, as highlighted below:

Girl 1: How much did you find on the third (problem)? (Addressed to all members)
Boy 1: We have not even gone to the third. (Boy 1answers).
(After a few minutes)...
Boy 1: I found "that much". Come on; find (a result) you guys!
(Tanya found a result)
Boy 1: Show me how much you found. (Addressed to Tanya)

School B

The 5th grade and the Mathematics class

In this class the learning atmosphere was excellent. The students worked cohesively in groups and the class performance level was high. The students were divided into four groups of four. These groups were generally stable, except on the second day when a boy was missing from the group under observation, and the teacher substituted a boy from another group. As for the mathematics class, the teacher on both tasked the students with solving revision problems.
The problems were given to the students printed on A4 paper, and simultaneously projected on the interactive whiteboard of the class. The teams solved the problems in the paper they had in front of them, and then they were called on the interactive whiteboard to show their solutions to the other groups. During the solution of problems to the whiteboard, the teacher asked clarifying questions to the students who solved the problem, and other groups were able to express their questions or comment on the proposed solutions.

1st day of observation

Course flow:

The teacher distributed to each group a problem common to all groups about the surface area. The students attempted to solve the problem. Then, the teacher checked the solutions the groups found, without commenting. The teacher then asked one members of each group to demonstrate, using the interactive whiteboard, the solution their group reached. After the presentation, the teacher asked the other groups if they had solved it in another way, or reached a different result. The group under observation had an opportunity to present its solution using the whiteboard.

As the students presented their solutions, the teacher asked questions and sought clarifications and explanations, both from the group which was presenting the solution, and from other students.

Participation and activity of the student in problem's solution

Both Girl 2 and Boy 2 actively participated in solving the problem. Boy 2 made more assumptions about the problem's solution and had a more active role than the rest of the group. Girl 2 made one assumption, and they were both justified their thinking aloud.
As a team, they solved the problem in 7 minutes.
As soon as they solved the problem, while waiting the other groups to finish, they discussed various things that concerned them beyond mathematics, with Boy 2 being more involved in the discussion than Girl 2.
During the presentation of the solution from another team, the team of the observation, as well as the rest of the teams, remained silent.
Both Girl 2 and Boy 2 (while solving the problem) often expressed their respective opinions.

Communication within the group

Both Girl 2 and Boy 2 cooperated very well with other team members. Girl 2 seemed to encourage the other members more than Boy 2 did. Girl 2 even asked for confirmation of her thoughts from the group, and expressed her opinion on a comment made by another member. Boy 2, on the other hand, twice expressed his queries to the other members of his team. Both Boy 2 and Girl 2 once completed the sentences of other team members.

The role of the group

The group under observation did not ask many questions. However, both times a query was expressed, the group solved it. In addition, twice it was observed that the members were discussing matters intensely and came to disagreement. The second time the dispute was small and involved the selection of the member who would get to the whiteboard to present the solution.

Communication between students and teacher

Both Girl 2 and Boy 2 asked once for a clarification from the teacher, while Girl 2 called the teacher to inform him that they had finished solving the problem. Three times, respectively, both Girl 2 and Boy 2 expressed their opinion on a comment made by the teacher, The first time concerned the solution of the problem (during problem solving), and the next two times were about the solution presented by another group.

Girl 2 twice raised queries with the teacher.

The use of language in communication

Both the whole group, and particularly Girl 2 and Boy 2, used "ordinary language" often, and, less often, used "mathematical spoken language." Once I noticed that Boy 2 used a "silent but common assumption" to progress a step in solving the problem.

2nd day of observation

Course flow:

The teacher checked the exercises the students had for homework, while raising questions and clarifications with individual students. The teacher then distributed the new problem, which also related to the surface area. The teacher read the problem to the class. At the same time, each group had a piece of paper with an image of the shape that describes the problem. The teams attempted to solve the problem, and then the teacher asked a representative from each group to present their solution on the interactive whiteboard. The teacher emphasized that the problem was not easy to solve. The group of the observation, however, got closest to solving the problem.

Participation and activity of the student in problem's solution

Both students, along with the entire group, participated fairly equally in solving the problem. Both made numerous assumptions about the solution of the problem and often justified their thinking, as well as frequently expressing their opinions to the group.

Communication within the group

Both Boy 2 and Girl 2, and the whole team, worked together very often. Boy 2, however, was the only one to encourage the other members of the group. They both raised questions with other group members, and gave their opinion on a comments made by other team members. Finally, Boy 2 and Girl 2 once completed the sentences - suggestions of other team members.

The role of the group

The team solved all the queries of its members, sometimes with intense discussion, but without coming to a disagreement.

Communication between students and teacher

Girl 2 asked once for clarification from the teacher, and twice she raised queries. Finally, she asked for his help once.

The use of language in communication

The use of "ordinary language" and “mathematical spoken language”, both by the whole team and particularly by the two students under observation, was common. Finally, "mathematical written language" was rarely used. However, when it was used, it was used more by the two research subjects, than by the other members of the group.

Interview from the students of the research

After the second day of observation, the students who participated in the research were asked to answer a few questions. In the question as to whether they like to work in a group or if they prefer to work for themselves, the students from School A answered that they preferred working in groups, while students from School B responded that they prefer to work alone. The students of School A justified their answer by saying that in peer collaboration the work is done by all members helping each other and correcting their mistakes. The students of School B argued that when working individually, you can think and decide for yourself, and *"the others don't ask you questions all the time".*

The second question was about what it was that they liked the most from the group work they experienced during the days of observation. Boy 1 replied that he liked more cooperation among team members, and Girl 1 agreed with him. Boy 2 and Girl 2 both indicated that what they liked was the fact that the thoughts of the students of the group were like a "puzzle", with each "piece" contributing and combining to the solution of the problem.

The students were asked to what was the most difficult part of working in groups. The students of School A indicated that there was nothing making it difficult for them to work in group. Girl 2 of School B replied the behavior of some members of the team made it difficult. Boy 2 of School B indicated that what made it difficult for him was that other group members demanded his attention for anything they were thinking. He referred to the tendency of other members to ask him to listen to and evaluate their suggestions *("listen to this", "here's one").*

In the last question of the interview about what they think they gained by working in groups, the students of School A answered that they had gained a better understanding of working and collaborating with their peers. The students of School B, completing each other's sentences, replied that group work allows *"new windows"* to open, such that *"you think in different ways and you know new ideas".*

DISCUSSION

In group problem solving, the observed groups generally followed the four steps suggested, among others, by Billstein, Libenskind & Lott, (1987), Schoenfeld, (1985) and de Corte et al, (2000), namely: a. understand the problem, b. design a plan b solution, c. execute the solution plan, and d. verification-confirmation. However, the group work differed from school to school. In School A students worked more individually, and in the end they shared the solution, while students at School B worked, from the outset, as a team. Initially some assumptions were presented, and at the end, the solutions were verified (Davidson, 1990). However, particularly in School A, more time was spent on the basics of checking correct arithmetic operations, so valuable time was wasted. Some students who could not keep up in math, such as Girl1, accepted their classmates' help (Davidson, 1990). In both classes of groups, members worked together and to some extent encouraged their peers (Davidson, 1990). Moreover, each group attempted to solve problems through a continuous dialogue with logical arguments (Davidson, 1990), some of which were accompanied with tension. Many times, especially in School B, students had entirely different approaches to solving problems, as indicated by subsequent discussions of the merits of the various proposed solutions (Davidson, 1990).

The language used by the students is consistent with the categories proposed by Pirie (1998). Most common was the use of ordinary language and the mathematical spoken and written language using symbols. School B, in particular, relied heavily on visual representation of both the data and the solution of the problems. Silent but common assumptions were rarely observed. What makes a great impression is that Girl1 was talking aloud to herself, as observed by Kotsopoulos (2010) in her research, for *"clarification of her thought".* The girl spoke aloud arithmetic operations she was doing while writing them, while none of the other group members considered it unusual behavior.

Finally, regarding communication between groups and the teachers of the research, it was observed that the students preferred to solve their queries first within their group. Only when students reached an impasse did they request the assistance of their teacher. This was found particularly in School B, where the teacher assisted the groups only when asked to. In School A, the teacher initially showed a similar behavior with the teacher of the second school, but during the second day she assisted with the work of groups without a request for assistance from any member. This intervention, to help some students with the solutions of the problems, resulted in some students moving to the next problems while the rest were left behind, as noted in the results section.

Of particular interest are the results of the interview. While it appeared that students of School A did not endorse working in groups (it is hypothesized this may be due to sub-optimal conditions for completing group work), they still recognized benefits and positive aspects of working with other students. The benefits of teamwork emerged especially from the students of School B. Initially, they compared the work in group with a “*puzzle*”, where the pieces are the thoughts of the team members, and all together make up the solution of the problem; with the effect that group work opens *"new windows"*, and *"different ways of thinking and new ideas".*

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APPENDIX

Date:
Grade:
School:
Students' names:

N/A **OBSERVATION DATA**	STUDENT A						STUDENT B					
Participation and activity of the student in problem solving												
1. Participates in the problem's solution												
2. Makes assumptions for the problem's solution												
3. Justifies his/her train of thought												
4. He/ She is competitive												
5. Interrupts the others												
6. He/ She is engaged to something else												
7. He/ She is silent (unproductive silence)												
8. Withdraws from the activity												
9. Expresses his/ her opinion												
Communication within the group												
10. Cooperates with the rest of the members of the team												
11. Encourages the other members												
12. Asks for support- confirmation of his/ her thinking from the other members of the team												
13. Expresses his/ her queries to the other members												
14. Expresses his/ her opinion about a comment outlined by another member of the team												
15. Completes the sentences- suggestions of the other members of the group												
The role of the group												
16. The group answers the queries of the other members												
17. The members discuss intensely												
18. The members come to a disagreement												
Communication between students and teacher												
19. The student asks for clarifications from the teacher												
20. The student expresses his/ her opinion for comments made by the teacher												

21. Expresses his/ her queries to the teacher												
22. Asks for help from the teacher												
The use of language in communication												
23. The student uses the “ordinary language”												
24. The student uses the “mathematical verbal language”												
25. The student uses the “mathematical written language”												
26. The student uses the “unspoken but shared assumptions”												
27. The student uses the “quasi- mathematical language”												
28. Forms and uses his/ her own words to attribute the problem’s terms												
29. Phrases frequently used by the students												