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Research Article

Free Vibration Analysis of a Thin-Walled Beam with Shear Sensitive Material

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This paper presents a method for a free vibration analysis of a thin-walled beam of doubly asymmetric cross section filled with shear sensitive material. In the study, first of all, a dynamic transfer matrix method was obtained for planar shear flexure and torsional motion. Then, uncoupled angular frequencies were obtained by using dynamic element transfer matrices and boundary conditions. Coupled frequencies were obtained by the well-known two-dimensional approaches. At the end of the study, a sample taken from the literature was solved, and the results were evaluated in order to test the convenience of the method.

1. Introduction

In the last two decades research on the dynamics of beams has grown enormously. There are numerous studies [1–29] on the bending-torsion coupled beam. In the beams, the elastic center and the center of mass are not coincident, so the translational and torsional modes are inherently coupled as a result of this offset. Rafezy and Howson [24] proposed an exact dynamic stiffness matrix approach for the three-dimensional, bimaterial beam of doubly asymmetric cross-section. The beam comprises a thin-walled outer layer that encloses and works compositely with its shear sensitive core material.

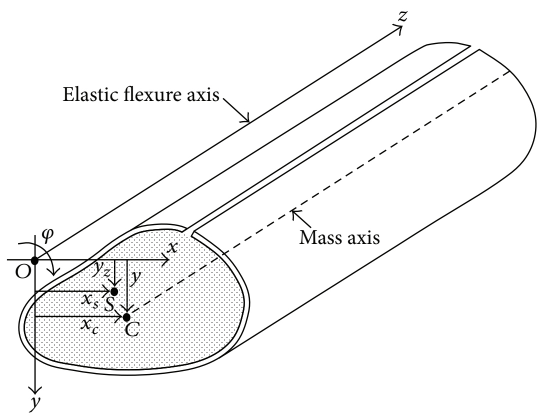
A dynamic transfer matrix method for the free vibration analysis of a thin-walled beam of doubly asymmetric cross-section filled with shear sensitive materials is suggested in this study. The following assumptions are made in this study: the behaviour of the material is linear elastic, small displacement theory is valid, and the dynamic coupling effect of structure caused by the eccentricity between the center of shear rigidity and the flexural rigidity center is ignored in analysis.

2. Analysis

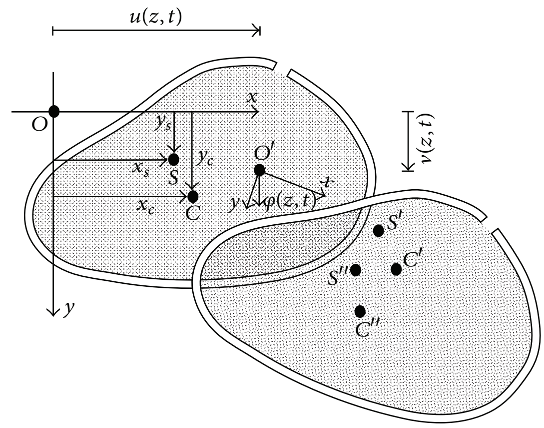
2.1. Physical Model. Figure 1 shows a uniform, three dimensional beam of length L. It has a doubly asymmetric cross-section comprising a thin-walled outer layer that encloses shear sensitive material [24]. The outer layer may have the form of an open or closed section that is assumed to provide warping and Saint-Venant rigidity, while the core materials provide Saint-Venant and shear rigidity. These assumptions lead to a model in which a typical cross-section has independent centers of flexure, shear, and mass denoted by O, S, and C, respectively [24]. For convenience, the origin of the coordinate system is located at the centre of flexure. O gives the result that the axis of elastic flexure coincides with the z-axis of the member. The x- and y-axes are subsequently aligned with the principle axes of the cross-section. The locations of the points S and C in the coordinate system Oxy are given by $S(x_s, y_s)$ and $C(x_c, y_c)$, respectively. The resulting elastic shear and mass axes then run parallel to the z-axis through (x_s, y_s) and (x_c, y_c) respectively. When the elastic axis of the beam does not coincide, the lateral and torsional motion of the beam will always be coupled in one or more planes [24].

2.2. Element Transfer Matrices for Planar Motion. The governing equations for ith element of uncoupled thin-walled shear sensitive beam can be written as

$$\rho_i \frac{\partial^2 U_i(z_i,t)}{\partial t^2} + EI_{xi}\frac{\partial^4 U_i(z_i,t)}{\partial z_i^4} - (GA)_{xi}\frac{\partial^2 U_i(z_i,t)}{\partial z_i^2} = 0,$$



(a)



(b)

Figure 1: Typical thin-walled beam [24].

$$\rho_i \frac{\partial^2 V_i(z_i,t)}{\partial t^2} + EI_{yi}\frac{\partial^4 V_i(z_i,t)}{\partial z_i^4} - (GA)_{yi}\frac{\partial^2 V_i(z_i,t)}{\partial z_i^2} = 0,$$
$$\rho_i r_m^2 \frac{\partial^2 \Psi_i(z_i,t)}{\partial t^2} + EI_{wi}\frac{\partial^4 \Psi_i(z_i,t)}{\partial z_i^4} - (GJ)_{oi}\frac{\partial^2 \Psi_i(z_i,t)}{\partial z_i^2} = 0, \tag{1}$$

where $GJ_{oi} = G_{ti}J_{ti} + GJ_{ci}$.

EI_{xi} and EI_{yi} are the flexural rigidity of the ith segment in the x-z and y-z planes, respectively, and $G_{ti}J_{ti}$ and EI_{wi} are the Saint-Venant and warping torsion rigidity of the ith segment about O, where I_w is the warping moment of inertia or warping constant. GA_{xi} and GA_{yi} are the effective shear rigidities of the core material of the ith segment in x and y directions, respectively, and GJ_{oi} is the Saint-Venant torsional rigidity of the core material about O. ρ_i are the mass per unit length of the ith segment, and r_m is the polar mass radius of gyration of cross section [24].

If a sinusoidal variation of U, V, and ψ with circular frequency ω is assumed then

$$U_i(z_i,t) = u_i \sin(\omega_x t),$$
$$V_i(z_i,t) = v_i \sin(\omega_y t), \tag{2}$$
$$\psi_i(z_i,t) = \theta_i \sin(\omega_\theta t),$$

where u_i, v_i, and θ_i are the amplitudes of the sinusoidally varying displacement.

Substituting (2) in (1) results are

$$\frac{d^4u_i}{dz_i^4} - \frac{(GA)_{xi}}{(EI)_{xi}}\frac{d^2u_i}{dz_i^2} - \frac{p_i}{(EI)_{xi}}\omega_x^2 u_i = 0,$$
$$\frac{d^4v_i}{dz_i^4} - \frac{(GA)_{yi}}{(EI)_{yi}}\frac{d^2v_i}{dz_i^2} - \frac{p_i}{(EI)_{yi}}\omega_y^2 v_i = 0, \tag{3}$$
$$\frac{d^4\theta_i}{dz_i^4} - \frac{(GJ)_{oi}}{(EI)_{wi}}\frac{d^2\theta_i}{d\theta_i^2} - \frac{p_i r_m^2}{(EI)_{wi}}\omega_\theta^2 \theta_i = 0.$$

When (3) is solved with respect to z_i, $u_i(z_i)$, $v_i(z_i)$, and $\theta_i(z_i)$ can be obtained as follows:

$$u_i(z_i) = c_1\cosh(a_{xi}z_i) + c_2\sinh(a_{xi}z_i) + c_3\cos(b_{xi}z_i) + c_4\sin(b_{xi}z_i), \tag{4}$$

$$v_i(z_i) = c_5\cosh(a_{yi}z_i) + c_6\sinh(a_{yi}z_i) + c_7\cos(b_{yi}z_i) + c_8\sin(b_{yi}z_i), \tag{5}$$

$$\theta_i(z_i) = c_9\cosh(a_{\theta i}z_i) + c_{10}\sinh(a_{\theta i}z_i) + c_{11}\cos(b_{\theta i}z_i) + c_{12}\sin(b_{\theta i}z_i), \tag{6}$$

where α_{xi}, α_{yi}, $\alpha_{\theta i}$, b_{xi}, b_{yi}, and $b_{\theta i}$ can be calculated as follows:

$$a_{xi} = \sqrt{\frac{s_{xi}+p_{xi}}{2}}, \qquad a_{yi} = \sqrt{\frac{s_{yi}+p_{yi}}{2}},$$
$$a_{\theta i} = \sqrt{\frac{s_{\theta i}+p_{\theta i}}{2}}, \qquad b_{xi} = \sqrt{\frac{-s_{xi}+p_{xi}}{2}},$$
$$b_{yi} = \sqrt{\frac{-s_{yi}+p_{yi}}{2}}, \qquad b_{\theta i} = \sqrt{\frac{-s_{\theta i}+p_{\theta i}}{2}},$$
$$p_{xi} = \sqrt{\left(\frac{GA_{xi}}{EI_{xi}}\right)^2 + 4 * \frac{p_i\omega_x^2}{(EI)_{xi}}},$$
$$p_{yi} = \sqrt{\left(\frac{GA_{yi}}{EI_{yi}}\right)^2 + 4 * \frac{p_i\omega_y^2}{(EI)_{yi}}},$$

$$p_{\theta i} = \sqrt{\left(\frac{GJ_{oi}}{EI_{wi}}\right)^2 + 4 * \frac{p_i \omega_\theta^2}{(EI)_{wi}}},$$

$$s_{xi} = \frac{(GA)_{xi}}{(EI)_{xi}}, \quad s_{yi} = \frac{(GA)_{yi}}{(EI)_{yi}}, \quad s_{\theta i} = \frac{(GJ)_{oi}}{(EI)_{wi}}. \tag{7}$$

By using (4), (5), and (6), the rotation angles in x and y directions (u_i', v_i'), rate of twist (θ_i'), bending moments in x and y directions (M_{xi}, M_{yi}) and bimoment (M_{wi}), shear forces in x and y directions (V_{xi}, V_{yi}), and torque (M_{ti}) for ith element can be obtained as follows:

$$\begin{aligned}\frac{du_i(z_i)}{dz_i} &= c_1 a_{xi}\sinh(a_{xi}z_i) + c_2 a_{xi}\cosh(a_{xi}z_i) \\ &\quad - c_3 b_{xi}\sin(b_{xi}z_i) + c_4 b_{xi}\cos(b_{xi}z_i),\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{dv_i(z_i)}{dz_i} &= c_5 a_{yi}\sinh(a_{yi}z_i) + c_6 a_{yi}\cosh(a_{yi}z_i) \\ &\quad - c_7 b_{yi}\sin(b_{yi}z_i) + c_8 b_{yi}\cos(b_{yi}z_i),\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{d\theta_i(z_i)}{dz_i} &= c_9 a_{\theta i}\sinh(a_{\theta i}z_i) + c_{10} a_{\theta i}\cosh(a_{\theta i}z_i) \\ &\quad - c_{11} b_{\theta i}\sin(b_{\theta i}z_i) + c_{12} b_{\theta i}\cos(b_{\theta i}z_i),\end{aligned} \tag{10}$$

$$\begin{aligned}M_{xi}(z_i) &= EI_{xi}\frac{d^2u_i(z_i)}{dz_i^2} \\ &= EI_{xi}\left[c_1 a_{xi}^2\cosh(a_{xi}z_i) + c_2 a_{xi}^2\sinh(a_{xi}z_i)\right. \\ &\quad \left. - c_3 b_{xi}^2\cos(b_{xi}z_i) - c_4 b_{xi}^2\sin(b_{xi}z_i)\right],\end{aligned} \tag{11}$$

$$\begin{aligned}M_{yi}(z_i) &= EI_{yi}\frac{d^2v_i(z_i)}{dz_i^2} \\ &= EI_{yi}\left[c_5 a_{yi}^2\cosh(a_{yi}z_i) + c_6 a_{yi}^2\sinh(a_{yi}z_i)\right. \\ &\quad \left. - c_7 b_{yi}^2\cos(b_{yi}z_i) - c_8 b_{yi}^2\sin(b_{yi}z_i)\right],\end{aligned} \tag{12}$$

$$\begin{aligned}M_{wi}(z_i) &= EI_{wi}\frac{d^2\theta_i(z_i)}{dz_i^2} \\ &= EI_{wi}\left[c_9 a_{\theta i}^2\cosh(a_{\theta i}z_i) + c_{10} a_{\theta i}^2\sinh(a_{\theta i}z_i)\right. \\ &\quad \left. - c_{11} b_{\theta i}^2\cos(b_{\theta i}z_i) - c_{12} b_{\theta i}^2\sin(b_{\theta i}z_i)\right],\end{aligned} \tag{13}$$

$$\begin{aligned}V_{xi}(z_i) &= EI_{xi}\frac{d^3u_i(z_i)}{dz_i^3} - (GA)_{xi}\frac{du_i(z_i)}{dz_i} \\ &= \left[EI_{xi}a_{xi}^3\sinh(a_{xi}z_i) - (GA)_{xi}a_{xi}\sinh(a_{xi}z_i)\right]c_1 \\ &\quad + \left[EI_{xi}a_{xi}^3\cosh(a_{xi}z_i)\right. \\ &\qquad \left. - (GA)_{xi}a_{xi}\cosh(a_{xi}z_i)\right]c_2 \\ &\quad + \left[(EI)_{xi}b_{xi}^3\sin(b_{xi}z_i) + (GA)_{xi}b_{xi}\sin(b_{xi}z_i)\right]c_3 \\ &\quad + \left[-(EI)_{xi}b_{xi}^3\cos(b_{xi}z_i)\right. \\ &\qquad \left. - (GA)_{xi}b_{xi}\cos(b_{xi}z_i)\right]c_4,\end{aligned} \tag{14}$$

$$\begin{aligned}V_{yi}(z_i) &= EI_{yi}\frac{d^3v_i(z_i)}{dz_i^3} - (GA)_{yi}\frac{dv_i(z_i)}{dz_i} \\ &= \left[EI_{yi}a_{yi}^3\sinh(a_{yi}z_i) - (GA)_{yi}a_{yi}\sinh(a_{yi}z_i)\right]c_5 \\ &\quad + \left[EI_{yi}a_{yi}^3\cosh(a_{yi}z_i)\right. \\ &\qquad \left. - (GA)_{yi}a_{yi}\cosh(a_{yi}z_i)\right]c_6 \\ &\quad + \left[(EI)_{yi}b_{yi}^3\sin(b_{yi}z_i) + (GA)_{yi}b_{yi}\sin(b_{yi}z_i)\right]c_7 \\ &\quad + \left[-(EI)_{yi}b_{yi}^3\cos(b_{yi}z_i)\right. \\ &\qquad \left. - (GA)_{yi}b_{yi}\cos(b_{yi}z_i)\right]c_8,\end{aligned} \tag{15}$$

$$\begin{aligned}M_{ti}(z_i) &= EI_{wi}\frac{d^3\theta_i(z_i)}{dz_i^3} - (GJ)_{oi}\frac{d\theta_i(z_i)}{dz_i} \\ &= \left[EI_{\theta i}a_{\theta i}^3\sinh(a_{\theta i}z_i) - (GJ)_{oi}a_{\theta i}\sinh(a_{\theta i}z_i)\right]c_9 \\ &\quad + \left[EI_{wi}a_{\theta i}^3\cosh(a_{\theta i}z_i)\right. \\ &\qquad \left. - (GJ)_{io}a_{\theta i}\cosh(a_{\theta i}z_i)\right]c_{10} \\ &\quad + \left[(EI)_{wi}b_{\theta i}^3\sin(b_{\theta i}z_i) + (GJ)_{oi}b_{\theta i}\sin(b_{\theta i}z_i)\right]c_{11} \\ &\quad + \left[-(EI)_{wi}b_{\theta i}^3\cos(b_{\theta i}z_i)\right. \\ &\qquad \left. - (GJ)_{oi}b_{yi}\cos(b_{\theta i}z_i)\right]c_{12}.\end{aligned} \tag{16}$$

The following equation shows the matrix form of (4), (8), (11), and (14):

$$\begin{bmatrix} u_i(z_i) \\ u_i'(z_i) \\ M_{xi}(z_i) \\ V_{xi}(z_i) \end{bmatrix} = A_{xi}(z_i)\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}. \tag{17}$$

For the y direction, the following shows the matrix form of (5), (9), (12), and (15):

$$\begin{bmatrix} v_i(z_i) \\ v_i'(z_i) \\ M_{yi}(z_i) \\ V_{yi}(z_i) \end{bmatrix} = A_{yi}(z_i) \begin{bmatrix} c_5 \\ c_6 \\ c_7 \\ c_8 \end{bmatrix}. \quad (18)$$

Similarly, torsional motion can be written:

$$\begin{bmatrix} \theta_i(z_i) \\ \theta_i'(z_i) \\ M_{wi}(z_i) \\ M_{ti}(z_i) \end{bmatrix} = A_{\theta i}(z_i) \begin{bmatrix} c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix}. \quad (19)$$

At the initial point of the ith element, (17), (18), and (19) can be written as follows:

$$\begin{bmatrix} u_i(0) \\ u_i'(0) \\ M_{xi}(0) \\ V_{xi}(0) \end{bmatrix} = A_{xi}(0) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}, \quad (20)$$

$$\begin{bmatrix} v_i(0) \\ v_i'(0) \\ M_{yi}(0) \\ V_{yi}(0) \end{bmatrix} = A_{yi}(0) \begin{bmatrix} c_5 \\ c_6 \\ c_7 \\ c_8 \end{bmatrix}, \quad (21)$$

$$\begin{bmatrix} \theta_i(0) \\ \theta_i'(0) \\ M_{wi}(0) \\ M_{ti}(0) \end{bmatrix} = A_{\theta i}(0) \begin{bmatrix} c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix}. \quad (22)$$

When vector c is solved from (20) and is substituted in (17), the following is obtained:

$$\begin{bmatrix} u_i(z_i) \\ u_i'(z_i) \\ M_{xi}(z_i) \\ V_{xi}(z_i) \end{bmatrix} = A_{xi}(z_i) A_{xi}(0)^{-1} \begin{bmatrix} u_i(0) \\ u_i'(0) \\ M_{xi}(0) \\ V_{xi}(0) \end{bmatrix}. \quad (23)$$

For $z_i = l_i$, (23) can be written as

$$\begin{bmatrix} u_i(l_i) \\ u_i'(l_i) \\ M_{xi}(l_i) \\ V_{xi}(l_i) \end{bmatrix} = T_{xi} \begin{bmatrix} u_i(0) \\ u_i'(0) \\ M_{xi}(0) \\ V_{xi}(0) \end{bmatrix}, \quad (24)$$

where T_{xi} is the element dynamic transfer matrix of the ith element.

For the y direction, (23) and (24) can be written as follows:

$$\begin{bmatrix} v_i(z_i) \\ v_i'(z_i) \\ M_{yi}(z_i) \\ V_{yi}(z_i) \end{bmatrix} = A_{yi}(z_i) A_{yi}(0)^{-1} \begin{bmatrix} v_i(0) \\ v_i'(0) \\ M_{yi}(0) \\ V_{yi}(0) \end{bmatrix}, \quad (25)$$

$$\begin{bmatrix} v_i(l_i) \\ v_i'(l_i) \\ M_{yi}(l_i) \\ V_{yi}(l_i) \end{bmatrix} = T_{yi} \begin{bmatrix} v_i(0) \\ v_i'(0) \\ M_{yi}(0) \\ V_{yi}(0) \end{bmatrix}. \quad (26)$$

Similarly, rotation motion can be written in equations as follows:

$$\begin{bmatrix} \theta_i(z_i) \\ \theta_i'(z_i) \\ M_{wi}(z_i) \\ M_{ti}(z_i) \end{bmatrix} = A_{\theta i}(z_i) A_{\theta i}(0)^{-1} \begin{bmatrix} \theta_i(0) \\ \theta_i'(0) \\ M_{wi}(0) \\ M_{ti}(0) \end{bmatrix}, \quad (27)$$

$$\begin{bmatrix} \theta_i(l_i) \\ \theta_i'(l_i) \\ M_{wi}(l_i) \\ M_{ti}(l_i) \end{bmatrix} = T_{\theta i} \begin{bmatrix} \theta_i(0) \\ \theta_i'(0) \\ M_{wi}(0) \\ M_{ti}(0) \end{bmatrix}. \quad (28)$$

If (24) is written successively, the displacements—internal forces relationship between the initial part and end of the beam—can be found as follows:

$$\begin{aligned} \begin{bmatrix} u_{\text{end}} \\ u'_{\text{end}} \\ M_{x\text{end}} \\ V_{x\text{end}} \end{bmatrix} &= T_{xn} T_{x(n-1)} \cdots T_{x2} T_{x1} \begin{bmatrix} u_{\text{initial}} \\ u'_{\text{initial}} \\ M_{x\text{initial}} \\ V_{x\text{initial}} \end{bmatrix} \\ &= t_x \begin{bmatrix} u_{\text{initial}} \\ u'_{\text{initial}} \\ M_{x\text{initial}} \\ V_{x\text{initial}} \end{bmatrix}. \end{aligned} \quad (29)$$

For y and rotation motion, (29) can be written as follows:

$$\begin{aligned} \begin{bmatrix} v_{\text{end}} \\ v'_{\text{end}} \\ M_{y\text{end}} \\ V_{y\text{end}} \end{bmatrix} &= T_{yn} T_{y(n-1)} \cdots T_{y2} T_{y1} \begin{bmatrix} v_{\text{initial}} \\ v'_{\text{initial}} \\ M_{y\text{initial}} \\ V_{y\text{initial}} \end{bmatrix} \\ &= t_y \begin{bmatrix} v_{\text{initial}} \\ v'_{\text{initial}} \\ M_{y\text{initial}} \\ V_{y\text{initial}} \end{bmatrix}, \\ \begin{bmatrix} \theta_{\text{end}} \\ \theta'_{\text{end}} \\ M_{w\text{end}} \\ M_{t\text{end}} \end{bmatrix} &= T_{\theta n} T_{\theta(n-1)} \cdots T_{\theta 2} T_{\theta 1} \begin{bmatrix} \theta_{\text{initial}} \\ \theta'_{\text{initial}} \\ M_{w\text{initial}} \\ M_{t\text{initial}} \end{bmatrix} \\ &= t_\theta \begin{bmatrix} \theta_{\text{initial}} \\ \theta'_{\text{initial}} \\ M_{w\text{initial}} \\ M_{t\text{initial}} \end{bmatrix}. \end{aligned} \quad (30)$$

The eigenvalue equation for a thin-walled beam filled with shear sensitive material can be established using (29), (30), and the specific boundary conditions are as follows.

(1) Clamped-Free: $f_x = t_x(3,3)^* t_x(4,4) - t_x(3,4)^* t_x(4,3) = 0$, $f_y = t_y(3,3)^* t_y(4,4) - t_y(3,4)^* t_y(4,3) = 0$, $f_\theta = t_\theta(3,3)^* t_\theta(4,4) - t_\theta(3,4)^* t_\theta(4,3) = 0$.

(2) Clamped-Clamped: $f_x = t_x(1,3)^* t_x(2,4) - t_x(1,4)^* t_x(2,3) = 0$, $f_y = t_y(1,3)^* t_y(2,4) - t_y(1,4)^* t_y(2,3) = 0$, $f_\theta = t_\theta(1,3)^* t_\theta(2,4) - t_\theta(1,4)^* t_\theta(2,3) = 0$.

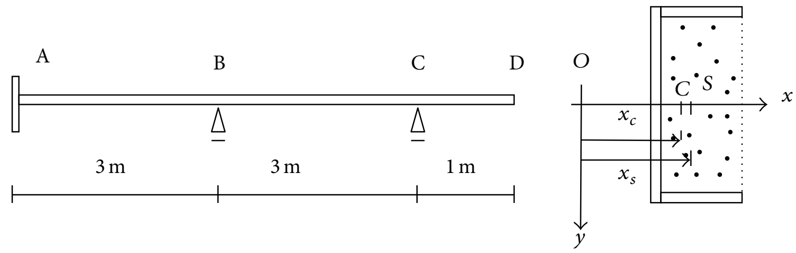


FIGURE 2: The doubly asymmetric, continuous channel section and the cross section of beam of example 2 with warping allowed at B, C, and D but fully constrained at A.

(3) Simply-Simply: $f_x = t_x(1,2)^* t_x(3,4) - t_x(3,2)^* t_x(1,4) = 0$, $f_y = t_y(1,2)^* t_y(3,4) - t_y(3,2)^* t_y(1,4) = 0$, $f_\theta = t_\theta(1,2)^* t_\theta(3,4) - t_\theta(3,2)^* t_\theta(1,4) = 0$.

(4) Free-Free: $f_x = t_x(3,1)^* t_x(4,2) - t_x(3,2)^* t_x(4,1) = 0$, $f_y = t_y(3,1)^* t_y(4,2) - t_y(3,2)^* t_y(4,1) = 0$, $f_\theta = t_\theta(3,1)^* t_\theta(4,2) - t_\theta(3,2)^* t_\theta(4,1) = 0$.

(5) Clamped-Simply: $f_x = t_x(1,3)^* t_x(3,4) - t_x(1,4)^* t_x(3,3) = 0$, $f_y = t_y(1,3)^* t_y(3,4) - t_y(1,4)^* t_y(3,3) = 0$, $f_\theta = t_\theta(1,3)^* t_\theta(3,4) - t_\theta(1,4)^* t_\theta(3,3) = 0$.

In frequency equations the values of ω, which set the determinant to zero, are the uncoupled angular frequencies.

2.3. Coupled Frequencies. Ignoring the dynamic coupling effect of structure caused by the eccentricity between the center of shear rigidity and the geometric center the coupled frequencies of the shear torsional beam can be obtained by using uncoupled frequencies and the well-known equation as follows [28]:

$$\begin{vmatrix} \omega_j^{(i)^2} - \omega_x^{(i)^2} & 0 & -y_c \omega_j^{(i)^2} \\ 0 & \omega_j^{(i)^2} - \omega_y^{(i)^2} & x_c \omega_j^{(i)^2} \\ -y_c \omega_j^{(i)^2} & x_c \omega_j^{(i)^2} & r_m^2 \left(\omega_j^{(i)^2} - \omega_\theta^{(i)^2}\right) \end{vmatrix} = 0 \qquad (31)$$

$$(j = 1, 2, 3) \quad (i = 1, 2, 3 \ldots).$$

3. Procedure of Computation

A program that considers the method presented in this study as a basis has been prepared in MATLAB, and the operation stages are presented below.

(1) element dynamic Transfer matrices are calculated for each element by using (24), (26), and (28).

(2) System dynamic transfer matrices (see (29)–(30)) are obtained with the help of element transfer matrices.

(3) The angular frequencies of uncoupled vibrations are obtained by using the boundary conditions.

(4) The coupled angular frequencies are found by using (31).

4. A Numerical Example

In this part of the study two numerical examples were solved by a program written in MATLAB to validate the presented method. The results are compared with those given in the literature.

4.1. Numerical Example 1. The first example considers the beam studied by Tanaka and Bercin [11]. A typical uniform thin-walled beam has a length of 1.5 m with a doubly asymmetric cross section. The properties of the cross section are as follows:

$x_c = 0.02316$, $y_c = 0.02625$, $\rho = 1.947$ kg/m, $r_m^2 = 3.0303^* 10^{-3}\,\text{m}^2$,

$EI_x = 73480\,\text{Nm}^2$, $EI_y = 16680\,\text{Nm}^2$, $EI_w = 23.64\,\text{Nm}^4$, and $GJ_0 = 10.81\,\text{Nm}^2$.

The first three coupled natural frequencies of the beam are calculated by the presented method and compared with the results by Tanaka and Bercin [11] and Rafezy and Howson [24] in Table 1 for clamped-free (C-F) and simply-simply (S-S) boundary conditions.

4.2. Numerical Example 2. A typical continuous beam with a doubly asymmetric cross section is considered in this example (Figure 2).

The beam comprises a thin-walled outer layer and a shear core with the following properties between support points A and B. The typical uniform thin-walled beam has a length of 1.5 m with a doubly asymmetric cross section. The properties of the cross section are as follows:

$x_s = 0.08$, $y_s = 0.03$, $x_c = 0.05$, $y_c = 0.02$, $\rho = 20$ kg/m, $r_m^2 = 0.008\,\text{m}^2$,

$EI_x = 2.16^* 10^6\,\text{Nm}^2$, $EI_y = 1.73^* 10^6\,\text{Nm}^2$, $G_t J_t = 3200\,\text{Nm}^2$,

$EI_w = 1.4^* 10^3\,\text{Nm}^4$, $GA_x = 600000$ N, $GA_y = 600000$ N, and $GJ_c = 3800\,\text{Nm}^2$.

The shear core is omitted between points B and D, where the cross-sectional properties remain unchanged, except that $GA_x = GA_y = GJ_c = 0$, and the small change in ρ has been ignored.

Table 1: Coupled natural frequencies for the beam of example 1.

BC	Natural frequencies (Hz)								
	Proposed method			Tanaka and Bercin [11]			Rafezy and Howson [24]		
	f_1	f_2	f_3	f_1	f_2	f_3	f_1	f_2	f_3
C-F	17.17	27.31	59.10	17.03	27.58	59.25	17.17	27.31	59.10
S-S	44.71	75.14	164.87	41.48	74.12	164.11	44.71	75.14	164.87

Table 2: Coupled natural frequencies of the continuous beam of example 2.

Frequency number	This study	Rafezy and Howson[24]	Difference (%)
1	6.906	6.940	−0.49
2	19.763	19.796	−0.17
3	35.461	33.836	4.80

The first three coupled natural frequencies of the beam are calculated by the presented method and compared with the results of Rafezy and Howson [24] in Table 2.

The main source of error between the proposed method and Rafezy and Howson methods is the eccentricity between the center of shear stiffness and flexural stiffness which was not taken into account in the proposed method.

5. Conclusions

This paper presents a method for a free vibration analysis of a thin-walled beam of doubly asymmetric cross section filled with shear sensitive material. In the study, first of all, a dynamic transfer matrix method was obtained for planar shear flexure and torsional motion. Then, uncoupled angular frequencies were obtained by using dynamic element transfer matrices and boundary conditions. Coupled frequencies were obtained by the well-known two-dimensional approaches. It was observed from the sample taken from the literature that the presented method gave sufficient results. The error margin of the proposed method is shown to be less than 5%. The main source of error is the eccentricity between the center of shear stiffness and flexural stiffness which was not taken into account in the proposed method.

The transfer matrix method is an efficient and computerized method which also provides a fast and practical solution since the dimension of the matrix for the elements and system never changes. Because of this the proposed method is simple and accurate enough to be used both at the concept design stage and for final analyses.

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